Making the learning of mathematical concepts a whole body -rather than just brain-involved experience
Maths in Motion team

Making the learning of mathematical concepts a whole body - rather than just brain - involved experience
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In May 2018, in a primary school in Italy, we did a little experiment. Our question was: how much do you like maths? The students replied with their bodies.

Before the Maths in Motion class

These “Dancing Snowflakes” used their bodies to explore rotational symmetric patterns, while their brains were learning.

After the Maths in Motion class

Werner (2001) mentioned that using the movement to approach mathematics, positively affects students’ attitude towards maths.

More than 250 students in 2018, who experienced Maths in Motion classes came to agree!
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This book is a result of the Maths in Motion project. It is a 3-year project (started on 2017), organized by Olde Vechte Foundation and funded by Erasmus+, under the KA2 Strategic Partnership for innovation. With this project, we bring together staff members of primary and secondary schools, enterprises and a dance school from 7 different partner organizations and countries, with the aim of developing an innovative approach to mathematics.

In 1966 Olde Vechte Foundation was created out of the initiative of a group of enthusiastic people who identified the need for learning which was not provided by the established educational system. The Foundation aimed to create a living and learning environment for individuals whose needs were not met by formal education. Olde Vechte Foundation is located in Ommen, the Netherlands and stands in the long and rich tradition that the country has in non-formal learning.

Society keeps on changing and it is becoming more and more intercultural. Still, the need of an “out of the school system” education remains. Olde Vechte provides a playground where people can experiment and experience different aspects of learning and development. The Foundation’s mission is to include everyone in its services regardless of age, gender, nationality and background. Like this, the Foundation brings to life its vision of love, care and cooperation.
The Maths in Motion project supports a cross-sectoral cooperation since it involves a diverse range of types of partners with different experiences, profiles and specific expertise. Mathematics and movement is a unique topic by itself and needs to be approached in a multidisciplinary way, in order to ensure project results. Hereby, we present them in the order they joined the project.

SciCo is a Non-Profit Organization dedicated to people’s engagement in science-related activities. The organization has been a pioneer in the relatively new and growing field of Science Communication in Greece. Since its foundation in 2008, they have been continuously developing their methods and techniques, gaining a widely recognized brand among institutions, schools, corporations and other stakeholders involved in education, science, and communication.

Fun Mathematics was founded as a social enterprise in 2011 in Sofia, Bulgaria. Following the motto “with heart, body and mind in maths”, its team spreads creative and joyful ways to approach mathematics. Fun Mathematics is an exhilaratingly inspired multilingual team with the mission “Play maths, touch maths, dance maths!”. Its main activity is an extracurricular program in mathematics for children in kindergarten and the primary school age, which aims to develop their critical thinking, confidence and ability for creative problem-solving in everyday situations.

Experience Workshop is an educational startup from Finland, which operates as a creative agency in the field of STEAM (Science, Technology, Engineering, Arts and Mathematics) learning. Experience Workshop is managing educational networks and developing projects to offer opportunities for children and people of all ages to learn mathematics through the arts and creative experiences, and to create art through mathematics. Its main areas of activities are: curriculum development, teacher education, project management, research, and organization of various educational events all over the globe. The Experience Workshop events’ methodological approach is based on cooperative, playful and experience-oriented problem-solving in mathematics and art learning, collaborative teaching and multi-, transdisciplinary, and phenomenon-based learning.

Scoala Gimnaziala Nr. 20 is situated in the outskirts of Galati, a city located in South-Eastern Romania. The school is governmental and non-profit, with the goal of educating pupils, age 6 to 14. The school has 6 primary classes and 4 secondary classes, educating approximately 250 pupils. The school’s mission is to be a place of pure thoughts for people looking to the future. Within this project, the school got active in Erasmus+ projects and now has become involved in the eTwinning platform.
Preface
A few words about our partners

Istituto Comprensivo is an Italian school and consists of three different levels of schools: kindergarten, primary school and secondary school. The Istituto Comprensivo educates approximately 1800 students and employs 171 teachers. The school teachers use creativity and the arts to develop and strengthen pupils’ skills. Since 2014 they have been part of a group of teachers who started to work on the “Math in Progress” project and they are also active in eTwinning.

The Ballet School in Holstebro was established in 1996 by the Royal Theater Ballet School in Copenhagen and is supported by The Danish Government, The Royal Danish Ballet and the Community of Holstebro. The Ballet School is private, and it is working on two levels: Dance and Academic. It is offering a space for students to practise their passion for dance and receive lessons in all normal subjects, as in any Danish public school. Moreover, the school is open for students with different challenges (academic, socially etc.) and children from foreign backgrounds.

Lena Nasiakou

Preface
Acknowledgements

Giving life to this toolkit is a result of much care, dedication, time and effort from a group of committed people. I would like to thank especially these people for their direct or indirect contribution to it.

To thank Olde Vechte Foundation for offering the space for the Maths in Motion project to be developed and to Marko Vlaming, the director of the Foundation, for seeing the potential in this project. His guidance and help have brought our learning and development to another level.

When I am writing “us” is because I refer to Despoina Rafailidou as well as myself. Despoina is the co-creator of the Maths in Motion project, and everything that has been developed under it. She is coordinating the whole project with a lot of care, creativity and smoothness. Despoina is the person who, from the very first moment, I understood I could count on, no matter what. Above all, I would like to thank her because she has honoured me with her friendship and cooperation.

Special thanks to my caring partner Endri Vrapi. Words are not enough to express how grateful I am for his constant loving encouragement and source of energy. Next comes my family who push me forward and support me unconditionally. Here I want to appreciate my sister Stella Nasiakou, who was by my side from the very first moment and brought a lot of inspiration with her creative ideas.

The Maths in Motion team is the core of this toolkit and I would like to thank everyone for what they have contributed. Svetlana Goranova, who got involved in the project in the very early stage. Together we developed educational material and trained more than 30 teachers on how to integrate movement in mathematics.

Lena Nasiakou
Kristof Fenyvesi who brought the theoretical and research background in what we are doing and he never loses an opportunity to make the project visible and spread the results further. Kristof contributed directly to this toolkit with a richness of references, as well as a theory chapter.

Monika Eftimova, Yiorgos Vagiakis, Svetlana Goranova, Marilena Andrikopoulou, Vasileana Kaludova, Theo Anagnostopoulos, Sacra Lehto, Kristof Fenyvesi, Natalia Ghidotti, Juha Kylönen, Luisa Lenta, Kerry Osborne, Patrizia Sguazzzi, Yordan Hodzhev, Hanne Derdaek, Rikka Kosola, Dida Isaa, Gitte Lausen, Tincuta Tufa, Tania Duda who created, implemented and wrote Maths in Motion modules. Their dedication and enthusiasm for the project are priceless.

Rikka Kosola, Kerry Osborne, Yiorgos Vagiakis and Marilena Andrikopoulou for finding copyright free music for the Modules and contacting the creators to receive permission for the pieces that were under copyrights. Special thanks to Rikka for adding her dancing and theatre experience in the project and for sharing it through her inspiring chapter in this toolkit. Many acknowledgements should go to Kerry for her commitment to editing this toolkit. Kerry improved a lot the language and also made brilliant content suggestions in order to bring more clarity to the texts.

Special thanks to Camelia Sidahmet, Giada Totaro, Balanescu Dorin and Marchis Marcel for visually capturing our Maths in Motion moments in Greece, Italy and Romania.

The music composers and creators Merja Soria and Antti Martikainen for supporting our project by allowing us to add their music as suggestion. As well as the Bensound.com website for the same reason.

Thanks to the talented team: Nikoleta Rafailidou, the toolkit designer who brought this toolkit from raw text into a finished product and Despoina Rafailidou who added her “Meraki” * in the production process.

Big thanks to the European Commission and the Dutch Agency for Erasmus+ for trusting the idea and financing the whole Maths in Motion project – whose this toolkit is part of.

I would like to express our gratitude to Dr. Karl Schaffer and Mr. Erik Stern, two of the most influential people in the field of maths/dance integration. Thank you for providing the foundation for our project and for the inspiration.

Lastly, I would like to thank myself, for supporting the Maths in Motion team in their creative process and for choosing to put together my learning in this toolkit.
It is my first and it is a heart-made one.

Lena Nasiakou
October 2018

*Meraki: (n.) This is a word that modern Greeks often use to describe what happens when you leave a piece of yourself (your soul, creativity, or love) in your work. When you love doing something, anything, so much that you put part of yourself into it.
Introduction

Let’s get it started

This is a toolkit about mathematics and movement. It explains the basic idea behind the embodied approach to maths and it offers 6 Maths in Motion modules—meaning, activities that include the whole body in the maths learning process.

Most probably, you are already now puzzled with the words “maths” and “motion” placed together. Or you are wondering why to bother ourselves with bringing motion to maths instruction? After all, what does movement have to do with maths? Science with embodied arts? I have to admit, we had the same questions when we first heard about it. Curiosity leads us to “play” with this topic and explore the links between the two fields. After lots of experience, experimentation and research, there is one thing that we can promise: Maths in Motion provides a different context. A context where maths can be interesting, active, engaging and fun to learn.

Wanna know how? Then keep going! And I am not writing, “keep reading” because this topic needs to be experienced. Put on the suggested music, grab someone and do the activities together.

Mathematics is one of the subjects that has most contributed to the educational failure of many students since 1988, as it is stated in papers from the Bridges Organization’s Conferences (Moerman and Allé, 2016). Results in PISA are proving the same. Students who need to learn mathematics are not coming away with the skills and knowledge required of them. Of equal importance, or maybe the root cause of the problem, is that students weak in maths are not confident and enthused about the subject (Schaffer, Stern and Kim, 2001). Moving is a way to raise up their self-confidence and get them enthusiastic about the subject. Moreover, using the body to approach mathematics, has been proven to have positive effects on a student’s attitude towards maths (Werner, 2001).
This toolkit is for you if you are a classroom teacher, mathematician, movement expert, educator, STEAM specialist, dance teacher, parent and/or simply curious to learn how to combine maths and movement. For simplicity reasons we will be referring to ‘you’, as you are the teacher. We use simple language plus the mathematical and movement terminology which is essential in making the ideas understood. As long you dive into it, there is no chance you won’t understand it, even if you don’t have a background in maths or movement. Our invitation is to use it as an inspiration for designing your own Maths in Motion activities.

The toolkit has been written after the first year of implementing movement in classrooms, and the first results are very promising. In a survey we conducted for 15 teachers, maths educators and STEAM experts who have participated in the Maths in Motion project:

- 100% increased their job satisfaction while teaching maths through movement
- 87% addressed underachievement in maths, especially when dealing with learners with different learning styles
- 93% enriched their professional expertise with this new innovative and inclusive approach
- 93% rated highly their abilities to bring variety into their teaching/training style, enabling all kinds of learners to develop creative and inclusive didactic skills
- 79% deepened their own mathematical knowledge, because of working with movement and
- 93% boosted their motivation for teaching maths with movement

Introduction
Embodied learning

73 of the 76 students who participated in one of the Maths in Motion classes, noted that they enjoyed this way of learning maths, a lot. This happens because of a Maths in Motion class:

- Brings each student into the centre of the educational process
- Is inclusive because it allows everyone to participate up to their level
- Is memorable because it is embodied
- Stimulates mathematical and critical thinking
- Develops social skills (communication, cooperation, negotiation)
- Boosts creativity

Our purpose with this toolkit is to model all our learning, to make the lessons transferable and available for everyone who wants to learn more and benefit from Maths in Motion.
Introduction

Let’s run through the chapters

We start with taking one step back and exploring the idea behind using movement in mathematics (1st chapter). The 2nd chapter comes to add some more depth in the theoretical frame. It is all about STEAM education and what researchers have found about the benefits of movement for cognitive purposes. Next, we share our four main principles which are the foundations of our work (3rd chapter). The 4th chapter is there to highlight the innovative point of view for this topic.

The actual modules, designed by the project’s team follow. The maths topics that we are approaching in the modules are: the sense of space; from 2D to 3D, patterns and mathematical operations (5th chapter).

After getting the first taste of what it actually means to do Maths in Motion with your students, we talk specifics about your role (6th & 7th chapter). The concepts introduced in these two chapters are illustrated with examples from the modules.

In the 8th chapter, you can find some info about the evaluation that has been done, so you know where all these numbers and statistics that are popping up in the text are coming from. The last chapter has the resources we found inspiration from and which supported us in the development of this project and set up the grounds for writing this toolkit (9th chapter).

Now that we have the introductions out of the way, continue into the first chapter to debate why to use movement in a maths class. Before this, let me give you a taste.

“It has been an exciting experience for me, and for my students that were surprised. They were doing mathematics while enjoying so much. It’s been a great way of including everyone,” noted an Italian teacher who participated in two of the core Maths in Motion activities.
Why movement?

We can’t change a problem with the same way we created it. (Albert Einstein)

Maths classes are sometimes difficult to be in. Difficult both for the teacher who has to deal with disengaged students and for the students who can’t get the concepts and receive low test results. We find ourselves trying to figure out how to solve this problem, and usually we come up with more theories, more explanations and more homework.

Albert Einstein said, “we can’t change a problem with the same way we created it”. With this thought in mind, we gave a shot of movement in maths, and it worked. We saw that body movement sessions have a long-term impact on learners, are engaging, interesting, enhance self-expression, allow the sense of initiative to grow and function as students reflection on their learning. The power of body movement comes from the fact that it is a universal language which brings us back to the basics. Through body movement, we overcome obstacles caused by differences in language, nationality, religion, gender, culture, educational and social background.

Psychologists in the 1920s said that there are 3 types of learning styles and they developed the VAK (Visual-Auditory-Kinesthetic) Learning Styles Model by classifying the most common ways that people learn. According to the model, most of us prefer to learn in one of the three ways: visual, auditory or kinesthetic. A visually-dominant learner absorbs and retains information better when it is presented in, for example, pictures, diagrams and charts. An auditory-dominant learner prefers listening to what is being presented. They respond best to voices, for example, in a lecture or group discussion. A kinesthetic-dominant learner prefers a physical experience. They like a “hands-on” approach and respond well to being able to touch or feel an object or people. The traditional way of teaching mathematics supports the visual and the auditory learners.

Using movement supports individuals who learn kinesthetically to find an experiential base to help them understand (Schaffer, Stern and Kim, 2001). In a Maths in Motion class, kinesthetic, auditory and visual senses are involved so everybody can be included, fully participate, learn and benefit from it.

60 of the 76 students who participated in one of the Maths in Motion classes noted that the movement supported them to better understand the maths concepts, a lot.

There are more theories which prove that movement is great to use for cognitive reasons, and we will explore them in the next chapter. For now, we would like to clarify something, please don’t get us wrong. Movement is not a cure-all approach. It is a great way to work with students, but it has its own limitations and restrictions. At this point, it is critical to recognize that physical participation requires a far more complex, demanding educational style than just sitting, listening and talking, both for you and your students.
Learning mathematics is, above all, usually regarded as an intellectual activity. Consequently, some of the most repeated advice to a student, when it comes to mathematics, is “use your brain” or “stop for a while, and think!” Indeed, many times students are told to focus only on the task to be solved and expected to try to exclude all the external stimuli from their attention.

The general image of a mathematics class is that most of the students are sitting at their desks and desperately struggling with their task alone, a “brain in a vat”, as it is known from philosophical thought experiments. While other students still keep on moving, struggling with focusing their attention and so start “disturbing the class”.

Even a complex mathematical problem can be made easier to be solved, if its abstract components can be visualized or made concrete in some way. Fortunately, the available tools for demonstration, visualization and modeling are increasingly developed and more and more widely accessible and implemented in mathematics education. There is, however, a mathematical demonstration tool that is absolutely unique in its high level complexity, one that is always within your reach. This is the human body! Additionally, the human body has a feature that is not provided in any other mathematics demonstration tool. Your body is always directly connected to your brain!

If we look at the research results, that cognitive neuropsychology has to tell about the brain-body relationship and the body movement’s effects on learning, we can find the following statements supported in the research literature (AFIS, 2018):

• Physical activity may positively affect cognitive skills.
• Academic performance of physically active children can show several improvements.
• Regular physical activity might lead to structural and functional changes in the brain that can play a primary role in achieving better intellectual results.
• Integration of dance and movement in mathematics instruction for students diagnosed with learning disabilities/emotional/behavioral disorder/attention-deficits/hyperactive disorders, can play a key role in increasing their mathematical performance (Anderson, 2015).

And these might be among your reasons to move forward…

So, let’s not stop here, but ask: would it be possible to convert children’s kinetic energy to motivation in mathematics learning? Is it possible to make mathematical discoveries through body movement? Is it possible that motivation and engagement provided by the group movement activities can be useful in reducing negative emotions toward mathematics? (Rosenfeld, 2016)

To find out your answers to these questions might be also among the reasons to move forward.
Reasons to Move Forward

With the embodiment and movement-based learning strategies, both the development of Gardner’s multiple intelligences (Gardner, 1983) and Burnard’s multiple creativities (Burnard, 2016) can become your primary learning goals side-by-side. Structural, spatial, rhythmic and symbolic dimensions of mathematics can be experienced simultaneously through body movement. Multidisciplinary and multi-sensory learning open up new dimensions for students, teachers, and even parents (Gerofsky, 2011; Katai et al., 2014; Watson, 2005). Body movement might be able to bring you out from your comfort zone to find new, exciting and complex topics (Sung et al., 2017). Everything that movement and dance can offer (Renesse, 2018), e.g. implementing phenomenon-based approaches and personalized, experience-oriented, playful methods in the learning process, come to contribute to discovering the rich versatility of mathematics. Going on new adventures might be another reason to move forward.

Lena Nasiakou

Principles

“Nothing is learned without the learner interacting with what is to be learned” (Moerman & Alle, 2016)

With the embodiment and movement-based learning strategies, both the development of Gardner’s multiple intelligences (Gardner, 1983) and Burnard’s multiple creativities (Burnard, 2016) can become your primary learning goals side-by-side. Structural, spatial, rhythmic and symbolic dimensions of mathematics can be experienced simultaneously through body movement. Multidisciplinary and multi-sensory learning open up new dimensions for students, teachers, and even parents (Gerofsky, 2011; Katai et al., 2014; Watson, 2005). Body movement might be able to bring you out from your comfort zone to find new, exciting and complex topics (Sung et al., 2017). Everything that movement and dance can offer (Renesse, 2018), e.g. implementing phenomenon-based approaches and personalized, experience-oriented, playful methods in the learning process, come to contribute to discovering the rich versatility of mathematics. Going on new adventures might be another reason to move forward.

1. Questioning
In a Maths in Motion classroom, you as a teacher can question and change everything! First of all, the setup of the room, where you are standing, the atmosphere that you want to create. Plus your students can question everything, so they start looking at maths, not as a static science but as an ongoing discovery. While encouraging students to share their thoughts and questions, without pointing out if they are wrong or right, you support them to clarify their understanding and even teach you some new solutions.
2. Interaction
We see interaction as a pivotal part of teaching and learning. Interaction in a Maths in Motion class becomes a very dynamic and playful term and it happens on three levels: 1. between teacher and student, 2. between the student and their fellow classmates and 3. between the student and their understanding of maths. In such a class all three levels of interaction unfold interwoven. The interplay between teacher and learner, as well as peer to peer learning, brings up another part of interaction, in a body and mind level. Perhaps most importantly, the learner is interacting directly, physically and mentally with the maths concept and in such a manner Maths In Motion guides them to joyfully interact with what maths is/can be.

“Nothing is learned without the learner interacting with what is to be learned” (Moerman & Alle, 2016)

3. Growth Mindset
Struggles, mistakes and perseverance are part of the learning process instead of signs of maths failure. A Maths in Motion class is not a place for good and bad students, but a place for a learner to develop a growth mindset. The growth mindset is based on the belief that qualities can be cultivated. How to do so? Simply praise students for the process, their engagement in the process and their work!
4. Creative coaching
You know the maths subject, you have studied it, you have taught it many times. No doubt around. What if you don’t act as such? What if, you use creative coaching techniques instead of passing on ready-made knowledge. What if, you guide students in how to reason, reflect, discuss with each other, and evaluate their ideas. What if, you are there to support their independent thinking and doing?

Principles

"Nothing is learned without the learner interacting with what is to be learned" (Moerman & Alle, 2016)

Innovation

"Instead of thinking outside of the box, get rid of the box.” Deepak Chopra

We like to think that movement is an innovative approach to maths. Here are the reasons that support our thoughts:

Geographical Context
The combination of maths and movement has been approached extensively in the USA. There has been also done a lot of research in northern Europe about movement and as well as in combination with other art forms (drawing, installations etc). However very few parts of the research had been actually implemented in schools. Our approach is hands on. That means that aims to bring awareness of the combination of maths and movement to the European environment through creating and implementing Maths in Motion modules to students.

Subject matter
Maths as a subject matter is usually approached by the book approach (traditional way of teaching). Sometimes it is also approached in a more dynamic way with games, technology and/or connected with life events (shopping, making discounts etc). Incorporating movement brings about more interesting, enjoyable and holistic ways to learn maths since it involves all the senses.
**Inclusive**

It is a way to attract learners that are usually signing off from the educational process when it comes to maths. Imagine a hyperactive student, a kinesthetic learner, a child with learning difficulties. Because of their nature, they are quite excluded from traditional learning procedures. With movement, learning maths gets another context which is inviting and dynamic, it engages all kinds of learners in the learning process.

55 of the 76 students who participated in one of the Maths in Motion classes noted that they “felt the team spirit” during their participation, a lot.

**Organizations and people**

There is innovation in the combination of the organisations. They are coming from different fields: formal and non-formal, different levels of education, enterprises and arts and from different countries: the Netherlands, Greece, Bulgaria, Finland, Denmark, Romania and Italy.

Moreover, the combination of people, the Maths in Motion team, is innovative. The project team is comprised of: experts in Mathematics, in non-formal education and body movement, people from the business world, experts in alternative teaching and dance, general education teachers, STEAM education experts, specialists in Science communication Activities- with theatre and dance, dancers and dance teachers.
Who created the modules?

This chapter includes 6 Maths in Motion modules. The modules have
been created by multidisciplinary teams, each of which consisted of
teachers, mathematicians, movement and STEAM experts and dancers.
The whole creation process could be defined as a cross-sectoral
cooperation since it involves quite a diverse range of professionals with
different experiences, profiles and specific expertise.

Every module has been designed for implementation in a specific
environment and for a specific target. From kindergarten to primary or
secondary schools, in Greece, Italy or Romania. The implementation of
the modules, supported us to see how they work in actual times with
various students. Out of this experience, we made corrections and
improvements and we are happy to share their final versions.

Each module follows a certain structure: starts with the maths topic, the
age group, the duration, the list of materials and music and the goals of
the module. The warm-up, the main body and the closing follow.

Lena Nasiakou
How and when to use the modules?

The modules come with an invitation for you to use them in your classroom. Mind that modifications and differentiation are always needed in order to help you cater them to the needs of your students. As for the music, we suggest some copyright free tracks that suit well the exercises. Feel free to choose your own music, to reorder or pick a different set.

The modules are intended to supplement the curriculum. That means that you can use them in different ways such as:

• At the beginning of a chapter as a kinesthetic introduction to the maths concept. As such, a set of maths concepts is introduced in a rather intuitive way and later on are revisited and reconstructed in a more formal way (Bruner, 1960 at Liao, 2012)

• At the end of a chapter, as a recap/summary. Like this, the maths ideas are revised and the students have the opportunity to look at them in a refreshing and insightful way.

• Instead of a “by the book” lesson. Get inspired by the modules and create Maths in Motion lessons in which students use their bodies to explore maths concepts, while their brains are learning.

• During a “by the book” lesson. During your regular class, you can integrate a short activity, for the students to embody the insight. For example, when teaching angles you can ask your students to pretend they are traffic officers and strike poses with their hands to show right, acute and obtuse angles.

• In a PBL (Project Based Learning). There the students in small groups, come up with a maths and movement project and you support them in the build-up. The final result could be to show their project to the other groups and/or to the neighbouring class.

Our wish with these modules is to give you a clear picture of what Maths in Motion is and to offer inspiration for creating your own activities.

We have created a Facebook group specially for this project. If you decide to use something from this toolkit, let us know! Contact us by an email at info.oldevechte@gmail.com or share your thoughts at the Facebook group. We look forward to inspiring each other.

facebook.com/groups/MathsinMotion
12 of the 18 students who participated in the Get Angled class noted that they understood the classification of angles a lot better.
Get Angled
Types, properties and categories of triangles

Activities

_Warm up (10-15 min)

- The students stretch in the bottom level, e.g. gorilla hop, crab kick, etc.

Stretching:

Monika Eftimova, Yiorgos Vagiakis and Svetlana Goranova
Monika Eftimova, Yiorgos Vagiakis and Svetlana Goranova

Get Angled
Types, properties and categories of triangles

Activities

Warm up (10 - 15 min)

Secret triangles
• Each student chooses two others students without letting them know. The chosen ones should be relatively physically close to each other. While moving, each student pays attention that he/she is always standing at equal distances from their chosen students.
• The students start moving slowly and freely in the space.
• While students are moving you say “Freeze”, students stop and start pointing at their chosen ones. When you say “Go!”, all start moving again.

Idea for different warm up:
Shape Triangles
• Each student shapes a triangle with part of her/his body.
• You gradually increase the number of triangles to be shaped with the body until it is impossible for the students to assume a posture for the requested number of triangles.
• The students individually show the smallest triangle they can shape with their bodies.
• The students show the largest triangle they can shape with their bodies.

Monika Eftimova, Yiorgos Vagiakis and Svetlana Goranova

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Get Angled

Types, properties and categories of triangles

Activities

Main Body

Get Angled!

• Explain to the students that the room is divided into 4 types of zones: the acute angles zone, the obtuse angles zone, the right angles zone and the 180 degree zone.

• The students move their arms in front of their body to show an acute angle, an obtuse angle and a right angle.

• When the students move in the space from one zone to another, they acknowledge it with a change in their arms’ span.

• Mute the music and shout “Freeze”.

• From the freezing position 3 students try to form a triangle without changing the angle of their arms.

• Discussion: Explain that it’s actually difficult to get a triangle from three random angles.

Ask what property do the angles in a triangle need to fulfill. (The sum of the angles should be 180 degrees.)

Setting: Split the room into two (with a rope) for the acute and obtuse angle zones. Designate the corners of the room as the right angle zone. The rope in the middle as well as the walls will be the zone for 0 or 180 degree angles. The idea is that the students move from one designated zone to another changing the span of their arms while doing so, in order to form acute, obtuse or right angles respectively.

Monika Efimova, Yiorgos Vagiakis and Svetlana Goranova
Get Angled
Types, properties and categories of triangles

Activities

Main Body

Shape triangles with respect to angles

• Call three students who want to volunteer for the next activity.

• The rest of the students are going to observe another way of forming a triangle.

• Ask one of the volunteers to step on the rope splitting the room and to hold his/her hands wide open (parallel to the rope).

• Ask the rest of the students to guess the angle between his/her arms, while expecting an answer of 180 degrees.

• Tell the second volunteer to step next to the first one and form two angles, the sum of which will still be 180 degrees.

Upon a troublesome execution

• You can ask the rest of the students to help.

• If they can’t come up with a solution, then the facilitator can support them by asking questions so they can come up with their answers. (Examples of questions: If the two volunteers could split the 180 angle into two angels, how would you do it by using in total 3 hands? And by using in total 4 hands? What are the possible measurements of the angles obtained?)

• Ask for the third volunteer to go in between the other two, forcing them to reduce their angles in order to keep the overall sum of the three angles, which is 180 degrees.

• Explain that now the three volunteers have three angles, the sum of which adds up to 180 degrees. So, they can shape a triangle.

• The three volunteers form their triangle and present it to the rest of the students.

• The rest of the children form triplets and shape triangles using the same process.

• Short discussion: what did you find out?
Get Angled
Types, properties and categories of triangles

Activities
Main Body
Let's play with the isosceles triangle:

This could be a second lesson:
• You hand an elastic rope to the students in each newly formed triangle and tell them to wrap it around them. The students then dance, while flowing from one triangle shape to another.
• Mute the music and call “Freeze.” The students will stop dancing and discuss the various triangle shapes formed throughout the class.
• Tell the students to go closer to each other and hug. While they are in a hug ask them what maths object is it that they are currently forming when together, expecting an answer of point.
• Tell them to slowly expand the point into a triangle. Ask them what would be necessary for a triangle to be an isosceles, expecting an answer of two sides of equal length or even better at least two sides of equal length.
• The students in each triangle line up in a single line next to each other and one of them goes in the middle. “How will you know that one of you is exactly in the middle?” Expect them to answer by folding the rope around the middle person.

• You say: “Can you make an isosceles triangle using the rope?” The group experiments with movement. For example the middle student can start moving back and forth (perpendicular to the line), thus forming various isosceles triangles. Let students find out other ways.
• Discussion: how did you obtain the isosceles triangle? What is important to have? Why was the middle point so important?

Put on music so the students dance freely as isosceles triangles. In the end encourage them to explore the various triangles they can form with the elastic rope and their bodies.

Have a short discussion about the categories of the angles and the triangles. What was important to have in order to build them?
Get feedback from children on how they felt about the teamwork. Students do a demonstration of a very small acute angle to mean they didn’t feel their team worked well together, an obtuse angle meaning they did, and an arms wide open in a 180-degree straight angle to mean they and their teammates were super! What can be improved in the triple for the exercise to be smoother?

• Tell them to slowly expand the point into a triangle. Ask them what would be necessary for a triangle to be an isosceles, expecting an answer of two sides of equal length or even better at least two sides of equal length.
• The students in each triangle line up in a single line next to each other and one of them goes in the middle. “How will you know that one of you is exactly in the middle?” Expect them to answer by folding the rope around the middle person.

• Tell the students to go closer to each other and hug. While they are in a hug ask them what maths object is it that they are currently forming when together, expecting an answer of point.

Monika Eftimova, Yiorgos Vagiakis and Svetlana Goranova
Dance of the Shapes

Statistics

16 of the 18 students who participated in the Dance of the Shapes class noted they better understood the coordinate systems.

Monika Eftimova, Yiorgos Vagiakis and Svetlana Goranova

Target group (age, number)

13 - 15 years, up to 20 - 24 students

Duration & Place

45 - 60 min

Materials & Music

Materials:
- Rope/Masking tape for the line of symmetry

Music:
- Hey!, Benjamin Tissot, Bensound.com
- Cute, Benjamin Tissot, Bensound.com
- Ukulele, Benjamin Tissot, Bensound.com
- Jazzy frenchy, Benjamin Tissot, Bensound.com
- Element, Benjamin Tissot, Bensound.com

For this module you need an assistant for the demos.

Goals

- Give perception of the position of the shape on the plane and how a shape is being translated and mirrored
- Cooperative learning
- Teamwork & leadership
- Problem solving
- Creativity

Monika Eftimova, Yiorgos Vagiakis and Svetlana Goranova
Dance of the Shapes
Symmetry, Translation

Activities

-Warm up

Air Whiteboard

• Put on a lively music track. The students dance in a free manner around the space.
• The students use parts of their body (arm, finger, head, elbow, leg, etc.) to write their name in the air. Guide them through and show example of each body part.
• The students use parts of their body (arm, finger, head, elbow, leg, etc.) to draw a shape in the air.

Body supports

• The students pair up.
• The students in couples create a few postures with leaning on each other, e.g. side lift, lean walk, partner squat, etc. You can give demos to show the posture and inspire them.
• The couples make the postures, use their imagination to come up with different ones and move around the room.

-Mirroring in couples

• The couples sit down on the floor.
• You introduce the concept of mirroring and the concept of levels (down, medium, up) and the couples start mirroring.
• The students get up and try the mirroring exercise using their legs. Like this they can experiment with a wider range of moves. As time passes by they can experiment in levels, by the encouragement of the trainer.

Monika Eftimova, Yiorgos Vagiakis and Svetlana Goranova
Dance of the Shapes
Symmetry, Translation

• You prepare in advance several coordinate systems on the floor with -x and -y coordinate lines.

• Students split into pairs. Couples get x and y coordinate of a point in the coordinate plane. They meet at the exact place of the point. Important: "x" should always start walking first to show that he is "x" the first coordinate. The y-student extend their arms and point at their respective x-kids.

• Switch to "one student - one point". Another student is the image of the point according the symmetry axes X. Students move similar to the warm up, but pay attention to the position on the coordinate plane. Then they switch roles.

• Then each couple forms a section (a line which has a beginning and an end). And another two students are the images of this section according the X-axes, one student in a couple mirrors one student in the other couple, so we get a movement of sections.

Monika Eftimova, Yiorgos Vagiakis and Svetlana Goranova

Dance of the Shapes
Symmetry, Translation

• Students are split into groups of three. A triple mirrors a triple with respect to X-axes. Members in the triple are invited to move from a point with whole coordinates to another point with whole coordinates, so that their mirrored images are with whole coordinates too.

• Another triple is now mirroring the first 3 students according to symmetry axes Y. You question the students: How are the 3 groups related to each other? There could be a fourth group that is mirroring the third group.

• In the end we receive a central symmetry. When the students hear: "Pose", they should strike a pose. "Switch", the mirroring and mirrored would switch. "Step to the front/back", all groups make a step in the given direction.

Monika Eftimova, Yiorgos Vagiakis and Svetlana Goranova
Dance of the Shapes
Symmetry, Translation

Activities

Main Body I

Dance of the shapes

• Two triples work as a team and prepare a sequence of 5-10 moves which they will perform together. Give them a designated time for that. You can use a beamer in order to show a countdown.
• Each triple present their dance and the corresponding triple on the other side would mirror them. After which the roles are reversed and the mirroring group presents their dance.

Monika Eftimova, Yiorgos Vagiakis and Svetlana Goranova

Closing Discussion

The students shape a circle and sit down. You can ask the students how they liked the exercise and what would their general feedback be. Here are some questions to support them give feedback:
• What did you learn from the activities?
• What was easier - to lead or to follow?
• Did you learn more about the coordinates and their mirrored image?

Created by Svetlana Goranova, Vasilena Kaludova, Monika Eftimova, Theo Anagnostopoulos, Yiorgos Vagiakis, Marilena Andrikopoulou
Composed by Monika Eftimova, Yiorgos Vagiakis and Svetlana Goranova. Photos by Despoina Rafalidou and Camelia Sidahmet
Edited and reviewed by Lena Nasiakou, Kerry Osborne and Svetlana Goranova
Dancing Snowflakes
Rotational symmetries

9+ years (the module works great with adults too), max. 20 participants
90 min, Sports hall / Large space for moving, Classroom

Target group (age, number)
Duration & Place

Materials
• Camera to record the snowflake dances
• Pictures of symmetric shapes, mandalas, snowflakes etc.
• 2D and 3D symmetry puzzles and construction kits

Materials & Music

Music
Calm meditative melodic music suggestions to be used during the whole module.

• Creation of the World, Antti Martikainen
• Amulla Varhain, Merija Soria
• Yksi, Kaksi, Kolme, Nelja, Merija Soria
• Sweet, Benjamin Tissot, Bensound.com
• Tenderness, Benjamin Tissot, Bensound.com

Goals
• Learn about rotational symmetric patterns through group movement activities
• Coordination of movement
• Empowering creativity
• Cooperative learning
• Problem solving
• Learning by doing
• Perceiving geometrical shapes through your own body
• Learning about different patterns of body movement
• Expand students’ ideas of ‘What is mathematics?’
• Encourage students to express their emotions through movement and monitor their feelings towards maths
• Reinforcing mathematical concepts in a cooperative kinesthetic manner

Kristóf Fenyvesi, Saara Lehto, Natalia Ghidotti, Juha Kylönen, Patrizia Sguazzi, Kerry Osborne
Dancing Snowflakes

Rotational symmetries

Activities

Warm up

Mirroring

• The students mirror the teacher’s moves (use slow and calm movements, so students can follow).

Kristóf Fenyvesi, Saara Lehto, Natalia Ghidotti, Juha Kyllönen, Patrizia Sguazzi, Kerry Osborne

• The students form couples and mirror their partners.

Kristóf Fenyvesi, Saara Lehto, Luisa Lenta, Natalia Ghidotti, Juha Kyllönen, Patrizia Sguazzi, Kerry Osborne
Dancing Snowflakes
Rotational symmetries

- The students form groups of 2-4 (depending on the number of participants). The students within each group experiment with their group members to discover the different kinds of bodily connections their group can make.
- Afterwards each group can show to others their favourite connection.
- Discuss different properties of the connections (big, small, symmetric, non-symmetric, low [on the floor], high, fun, brave, hands, legs, torso, head, facing each other, facing away from each other etc). Encourage students to use their own words, which can be linked to mathematical and artistic vocabulary by the teacher.
- What is also possible is that some groups might have achieved an inverted circle- to start in a circle holding hands and then without letting go, turn their circle ‘inside-out’. Within the discussion, such a move could be posed as a puzzle for the students to ponder how it may be achieved.
- Add to the discussion, ‘how do we communicate’ our intentions for others by our gestures or body movement, without directly asking.
- Discuss how many of the shapes created were symmetric even though that was not part of the original task.

The students, in groups of three, create different shapes on the floor: triangle, square, rectangle, circle, heart, star, fish (if there is time, create your own shape).

- The students divided into groups of 6 (or 5-8): sit on the floor, face each other in a circle, place their feet together and hold hands. Alternatively, stand facing each other and holding hands.
- They experiment with the kinds of symmetric shapes they can create in their group and they create shapes in their 6-folded symmetry system “snowflake.”
- The groups find ways to move from one shape into another. They decide on which order the movements should come and they need to remember their plan.

If a group is creatively stuck: i) Student 1 comes up with a connection and then the others copy one by one, then student 2 comes up with a movement and so on until the circle closes. Or ii) Divide into two equal sub-groups of three [A and B]. Experiment with the various kinds of movements and shapes you can make with everyone in group A doing the same thing and everyone in group B doing the same thing.

Kristóf Fenyvesi, Saara Lehto, Natalia Ghidotti, Juha Kylönen, Patrizia Sguazzi, Kerry Osborne
Based on all the ideas they come up with from above, each group creates a snowflake dance!

• All the groups perform their snowflake dances for one another.
• Film the snowflake dances from above (standing on a stool or a table), so the group performers can later see how their dance looks, too!
• Add a clip of their piece to a math portfolio the participants might have, and then have the group members work together to annotate the piece using mathematical terminology.

In the whole session, use slow and calm music, preferably without noticeable rhythm or melody so that the snowflakes can flow with their own pace. Music helps to create a nice atmosphere for the movement and for the performances.
### Activities

**Closing**

### Discussion on learning

### Further discussions

<table>
<thead>
<tr>
<th>Topics to talk about: snowflakes as 6-folded symmetric patterns, fractals, crystals, mandalas, symmetries, divisions of 6 (or 5 – 8, depending on the number of participants in the sub-groups), geometric shapes.</th>
</tr>
</thead>
<tbody>
<tr>
<td>• How the snowflake dances of 5 &amp; 7 kids versus 6 &amp; 8 kids looked different? Compare what kinds of things (movements, shapes) you can do with 5 in contrast to what is possible with 6 participants?</td>
</tr>
<tr>
<td>• If a maths portfolio is made, have the students do online searches to find where 5 or 7 patterns happen in nature (and where else other than snowflakes does 6 appear in nature?)</td>
</tr>
</tbody>
</table>

During the discussions, watching the videos can help students to reflect on what kinds of geometrical shapes the dancers created.

#### Follow-up activities

- Drawing snowflakes/mandalas of their own (alone or in groups, one student drawing something and others following making the whole picture symmetric). In addition to working on paper, drawing of mandalas also can be done outdoors on sand, 6 students drawing symmetric lines in the sand following each other.
- Use physical manipulatives, such as 2D and 3D puzzles and construction kits to model the patterns the group has created by movement.

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Kristóf Fenyvesi, Saara Lehto, Luisa Lenta, Natalia Ghidotti, Juha Kyllönen, Patrizia Sguazzi, Kerry Osborne
Monkeys and Elephants
Rhythmic patterns with movement, sounds and notations

Target group (age, number)
5+ years, max. 20 students

Duration & Place
90 min (or in several small sessions / longer time if the situation allows), Sports Hall

Materials
• Footprints of elephants and monkeys printed or drawn in large size on papers (find them at the end of the toolkit)

Music
• Song for a circle play in the language of the children. Make sure that the song is already well-known by the children
• Discover the rhythmical pattern of a song through movement.
• Experience the singing, the movement and the visual pattern together.
• Creatively build own patterns.
• Understand the difference between the long and short duration of a note.
• Acknowledge that some songs may vary from one singer to the next. By noting down how it is sung, everyone can sing it in the same manner.

Goals
• Transfer the singing & movement experience into a visual notation.
• Experience the singing, the movement and the visual pattern together.
• Creatively build own patterns.
• Understand the difference between the long and short duration of a note.
• Acknowledge that some songs may vary from one singer to the next. By noting down how it is sung, everyone can sing it in the same manner.

Activities
Warm up

Introduction
Discuss that you are going to move, dance, sing and learn mathematics together.

Circle dance
Singing and playing a circle dance, which is already well-known by the children.

Experiment with moving and walking in a circle like different animals—think:
• “What kind of animal is an elephant?” heavy and big with slow steps - to be attributed with the long duration of a note
• “...and the monkey?” fast and small with small steps- to be attributed with the short duration of a note
Sing the song as elephants stepping with the slow beat (dance and sing like an elephant) Sing the song as monkeys stepping with the fast beat (dance and sing like a monkey)

Kristóf Fenyvesi, Saara Lehto, Luisa Lenta, Natalia Ghidotti, Juha Kyllönen, Patrizia Sguazzi, Kerry Osborne
Monkeys and Elephants
Rhythmic patterns with movement, sounds and notations

Sing and dance the song again in a circle, now suggesting that the kids look carefully at the teacher’s feet and mimic the song with slow and fast steps according to the syllables in the lyrics.
Repeat until everyone understands the steps.

Sit down and discuss the different kinds of footprints various animals have. What kind of footprints do you think elephants have? What kind of footprints do the monkeys have? Show already prepared footprints of monkeys and elephants. Whose do you think is this and that?

Introduce papers with either one elephant footprint or two monkey footprints. Students can colour these footprints themselves!
Monkeys and Elephants
Rhythmic patterns with movement, sounds and notations

Build simple patterns “paths” of the footprints (e.g., monkey-monkey-elephant-elephant-monkey) and ask the students to experiment walking these paths by combining the monkey steps (fast) and elephant steps (slow).

Distribute footprints to the students (each having one footprint within their grasp). You build together the pattern of the song on the gym floor. You sing and dance the song together in small parts and build the path one footprint at a time. The students dance along the path. While moving, they can test to see if the footprints correlate with the song.
Activities

_Closing_

Walk, sing and dance the song path in canon. Next group starts after four beats.

Canon

Further suggestions

Divide the children into groups. Each group creates their own pattern. They then try each other’s patterns.

Introduce an easy repeating pattern, as a path: for example, monkey-monkey-elephant. Then divide the children into groups and ask them to make their own paths. They then try each other’s pattern paths.

Creating patterns on paper with colours or with legos etc.
Dragons’ World

Statics

19 of the 19 students who participated in the Dragons’ World class, felt that they better understood how to solve a problem.

Yordan Hodzhev, Hanne Derdau
Dragons’ World

Basic arithmetic, Problem solving

• Empowering creativity
• Cooperative learning
• Teamwork & leadership
• Problem solving
• Creation of own maths stories
• Exercise solving a problem in sequences of 2-3 maths operations
• Appreciation of one another and each of our achievements

Goals

This is a lesson with a magical element, and it requires a theatrical setting whether indoors or outdoors. Choose a specific place which will be your Magic Forest. The magical forest should be different from the classroom, and can hopefully be a place outdoors - ideally the school garden or park. This is the place where you will hide eggs. Put inside the eggs a candy, an animal a picture, a table with numbers and hide them well.

How to create a magic atmosphere in the forest? Decorate the Magic Forest accordingly for a more dramatic effect. Use markers (colourful/shiny pieces of paper or foil, wind wheels, garden decoration, etc.) to highlight the places where the eggs are.

Yordan Hodzhev, Hanne Derdau
You can completely transform yourself into a dragon with clothing, hat and tail. Practise the movements and the vocals of a dragon; this will support you in having the children believe you are one. Theatrical skills come in handy when leading the students and support them to overcome any moment of hesitation in participation.
Activity starts in the classroom. You hold a hula-hoop and say: "This hula-hoop is a door to a magic world". You go through the hula-hoop and the students do the same. When they are inside the teacher says "Once you’ve come through that door your eyes will soon see differently, your ears will hear differently and your skin will feel different." While saying it you gently massage the eyes, ears, skin respectively in order to having the children close their eyes and feel their human eyes morphing into dragon eyes, imagining their ears now taking a new form as dragon ears, and their being replaced by that of a dragons. "We are no longer human beings, we are now dragons!".
Saying this you transform your pose and movements, describing the students your dragon’s head/heads, elaborating on the colour of your skin, wings, tail and everything else needed to enter to the magical dragon world. Encourage the students to imaginarily turn into “real dragons”. Here are some possible questions to guide them:

- What kind of tail/eyes/wings do I have?
- How does my skin/tail/belly look?
- What kind of ears/mouth/smile do I have?
- How does my breath feel and smell?

The students move for a couple of minutes around with their new dragon bodies in order to feel comfortable in their new dragon skins. Whilst moving, you smoothly lead them to the Magic Forest.
Activities Part 2

Main body

Golden eggs

Dragon leaders

Newly born dragons want to explore the magic place. Raise the hula-hoop high, and ask, "Who wants to be a Dragon Leader?" Place the hula-hoop around a chosen dragon. All the other dragons need to hold onto the outside of the hula-hoop with one hand. Give them a map/card with instructions: "Now Dragon Leader, bring your dragon fellows to the first/biggest tree you see!" When dragons reach the tree, invite them to decide upon a new Dragon Leader and to fulfill the next objective - "Go to the nearest rock!". After the dragons get to the rock, the dragon group selects yet another Dragon Leader and they go to the last objective - "Go to the nearest bench!". Nothing has been found yet, this exercise is meant to get the children acquainted with the space and in the mood for a treasure hunt.

Now it is time for a real treasure hunt! Last night, Mother Dragon lost (20-use the same number as there are students) of her golden eggs. She went to look for food last night and when she came back, her nest was empty! Now each one of you dragons will need to search and find one egg.

The “treasure hunters” (students) go into the forest and you can wait in the middle of the Magic Forest and assign a special place where they can carefully place their fragile eggs. Ask questions to the dragons who find their eggs very quickly. The questions could be about their magical appearance as well as other quantitative questions:

- How many places did you go to before you found your egg?
- How many leaders have we had so far?
- How many ears, heads, eyes do you have?
- How many eggs would you lay as a dragon?

When all the dragons bring their eggs, they each open their own egg and find an egg candy and a note with a picture of an animal. Four groups are formed according to which animal they have a picture of. They gather into their groups. Thus, (with a group of 20) 4 groups are formed, each containing 5 students. Collect all the notes so they don’t get lost.
The classroom needs to be emptied of desks to create more space for movement. Sitting areas can be chairs along the walls or on the floor. The nest has 7 eggs, keep the rest for the next part. A drawing board or flipchart with markers is needed for the next part.

The next part is all about creating and solving maths stories. In order to enhance the problem solving skills of the students you can start with presenting a short pantomime maths story. Here is an example of pantomime. In the following example 3 more teachers participated.

While the students are sitting along the walls, one by one four dragons (teachers) each bring a magical gift egg to the nest. Each dragon approaches the nest with its own characteristic walk, to be sure the students are focused on them.

One might be a shy dragon, one a playful cartwheeling dragon, one a nervous suspicious dragon who is constantly looking around and startling themselves, one a moonwalking dragon or a yoga dragon...

Have fun be creative and make sure the kids are aware there are four dragons!

Yordan Hodzhev, Hanne Derdau
Activities Part 2

Main body

Story of the pantomime

The dragons put the 4 “gift eggs” into the nest (where 7 eggs are already—thus for a moment there are 11 eggs). The dragons are happy that they have their job done and start dancing. Suddenly, another animal (a teacher) approaches the nest and takes 3 of the eggs and runs away. The dancing dragons see what is going on and run to the nest but it is too late. They all show regret. End of the story.

Dragons’ World
Basic arithmetic, Problem solving

Discussion

Recreate the story

Discuss with the students about what mathematics they saw in the story. Support the students to see the pantomime as a maths problem with the following questions and put their answers on the whiteboard.

- How many eggs were there to begin with?
- How many dragons came to add a “gift egg”?
- How many eggs were there in the nest once the dragons added their “gift eggs”?
- How many eggs are left after the theft?
- How many eggs are missing?

Then the teacher asks the kids to recreate the story (construct the maths problem) out of those findings (7+4-3=8) and write it on the whiteboard/flipchart.
The students team up according to their groups formed previously at the end of the first part. Each group is gathered around the teacher who holds a picture of the group symbol. Each student receives a maths table, the same as they had found inside of their treasure hunt egg, and had given to the teacher before the break. On the back, they find maths symbols and numbers. The students play with the symbols (+, -, =) and magic numbers and they create mathematical sequences (e.g., 7+4-3=8).

Each group chooses a maths problem and devises a story around it. “Now your dragon group gets to create your own maths story and present it using movements.” (It can be either related to the teacher’s story - how they would help the dragons to find their missing eggs) or a completely different one.

The students work in groups; they plan and rehearse. You encourage the groups and support them, if needed.

Each group presents their story to the others. After each story, other groups try to decipher the maths behind the story and to create a math sequence out of the numbers, put it down and solve it. Applaud each of the groups.

Yordan Hodzhev, Hanne Derdau

Activities Part 2
Main body
New magic stories

Dragons’ World
Basic arithmetic, Problem solving

Diploma
Hold the hula-hoop and invite students to go back through the magic door and go back to the everyday world. As each dragon steps through the hula-hoop and enters the everyday world, they receive a dragon stamp testifying that they were explorers in a magical dragon’s world “for real”. Students receive a Diploma for successful completion of the challenge.

Evaluation
The students receive their eggs back and evaluate the session with them. If they are happy and enjoyed the activity, they put their dragon’s eggs back into the dragon’s nest (the hula-hoop). If they did not feel comfortable during the activities, they leave them outside the nest. You count the result and put the numbers on the whiteboard, as such: “total number, minus number of the eggs outside, equals the number of eggs within the nest.”
Negative Numbers, Positive Learning

Statics

Target group (age, number)
12 -13 years, 20 - 25 students

Duration & Place
50 min • 50 min

Materials & Music

Materials
• Laminated paper with numbers
• Painting tape
• Pencils to write on the tape
• Color markers
• Thread/string
• Paper
• Scissors

Music
• Music with elements evoking the feelings of “hot” and “cold”.
• “Hot” sound for music that has a fast tempo, increasing rhythm and which will make you rather sweaty if you literally follow the music’s pace, thus enhancing your imagination of what is hot and what it is to be in something hot.
• For “cold” sound for music that would be rather slow, or in decrescendo, in order to create a feeling of freezing. Something metallic and icy in the atmosphere for example.

21 of the 21 students who participated in the Negative numbers, positive learning class felt they had a better understanding of negative numbers.

Gitte Lausen
**Goals**

- Better understanding of and integration of the physical conceptualization of negative numbers through body movement and imaginary games.
- Demonstrating properties of negative numbers and fractions and expressing their emotional content.
- Empower teamwork and communication skills.

We all start in a circle and everyone in turn says his/her name. During the next round everyone says their name minus one letter ex. Gitte becomes Itte or Gitt or Gite.

**Activities**

**Warm up**

We continue in one or two rounds depending on the number of students and the situation. If it is a large group, the original circle is split into two circles for the second round.

You now introduce, with your hands, an imaginary shape belonging to your new name. You play with it for a couple of seconds, and then decide to turn either to your right or left so as to begin passing the shape around the circle in either a counter-clockwise or clockwise manner. You make eye contact with the person next to you and then give the shape away to the student. The student lets the imaginary object change into another shape and passes it along to the next person in the circle after having made eye contact with them.

When the shape is back to start, the second round starts. In this round the shape starts growing and growing while it is still passed on from one to another in the circle. On round three the imaginary shape is shrinking, still passing on from person to person. When it returns to you, it is now very small and you pretend to eat it.
Negative Numbers, Positive Learning
Negative numbers and communication by movement

On the fourth round you become whichever shape you so choose, and you do so with your whole body! Each person stands round in a circle in a static position. You begin by making a shape with your whole body—after it has been passed along, you will resume your static pose. As in previous rounds you pass the shape along to your neighbour. The shape forming passes in this manner all the way around the circle. When going around, the form changes every time from student to student.

Activities

_Warm up_

Lines

The students create two parallel lines, with equal numbers of people. You introduce the idea of creating big, small and neutral movements (be big, small or neutral with your body). Each student will create eye contact with the student from the opposite line, they exchange places and when they do that they have to be big or small (they have to decide by themselves using only body language). At the end we all take each others’ hands and move around the space like a long snake that finally turns into a kind of a line (to show a line of numbers it doesn’t have to be a straight line).
We give each student a piece of laminated paper with a number on it (from -12 to +12, remember 0). They can put their number on the floor in front of them, so they can see the line of numbers. One student comes off of the line and he or she will be the first tapper. That means that this student can tap other students. If a student gets tapped on the shoulder, the number of taps is positive and the tapped student grows bigger as (s)he travels up the number line the same number of spaces as (s)he was tapped. If they are tapped on the lower leg, the number is negative and they have to become smaller while they are moving towards the smaller numbers on the number line.
Warm and Cold

For the next part everyone finds a place in the room, there is music playing and the students decide whether it is a warm or cold kind of music. The students move around, to show how it feels to be warm or cold.

You play the cold music and they move around and show with their bodies that it is becoming colder and colder, at the end they freeze like statues. You ask them what their temperature is? And they are expected to give a negative number.

We move on in the same way, now with the "warm" music. Getting warmer and warmer, moving faster and faster, like molecules being warmed up. Then they end up melting and finally lying on the floor.

You move around and give every student a mark on the floor (painting tape) on which they can write their temperature (hot).

Teachers with physics and chemistry notions can add here what kind of scale we are talking about -- if it is degrees Celsius, or Kelvin or Fahrenheit, (or label and note down a specific molecule or element's melting point if they wish, or other relevant information).

You ask them if they can remember the spot where they froze and go back to it (if they cannot remember or want a new spot, they can move to another). They now get a piece of tape to mark the cold spot, and they write down the negative temperature. At the end, they have to imagine a line between their two spots and figure out where zero is.

The whole class can as well, reflect on questions such as:
- How many different measuring scales were used?
- Why do we use different measuring scales?
- Was each choice of measurement scale correct?
The student are placed into groups of 4. Each group of 4 is given some tools - painting tape, string, paper, markers/pencils, scissors.

Now they have to make their own exercise with negative numbers. Here we may add some colours in it too, and we should have some colour markers. We don’t mark colours before in order to let the students freely choose theirs in this part of the proposition, knowing that blue and red are commonly used to mark temperatures, for example.

You can have “extra-exercises and hints” in their pockets, how to use the tools for those who are quick and for those who struggle while finding their exercise. For example: by having a string tighten up between at least two students, how might one “limbo” the negative and positive scales in different ways in the space?

The students present for each other what they have created. We talk with the students about the module and what they have experienced and what they think and feel about it. In the end they demonstrate their evaluation through body movements, getting bigger or smaller. If not happy with their proposition, they make their bodies as small as their dissatisfaction, or if happy with their proposition, they demonstrate just how big and happy and pleased they are with such positive learning.
What is my role as a teacher?
Percentages, percentages, percentages

Now that you've gotten the first taste of embodied maths activities, let's dive more into your role. Needless to say, that is different than being a traditional teacher who possesses knowledge and skills, and lectures about them. There is not a single word to define this role, so I will use percentages: you are 26.4% organizer, 23.1% learning specialist, 17.7% atmosphere creator and 32.8% coach.

Organizer
You are the one who designs the content, organizes the lesson and prepares the materials. You are providing the context where the learning can happen. How to achieve this? By preparing activities and allowing the students to explore them. In other words, lay back and let them work. Imagine yourself as a stage director and enjoy the view from backstage. Ok, sometimes you will need to intervene, the following percentages are here to explain how. Don’t forget, a good organizer is sharp with times and safety issues.

Learning Specialist
There are some times when you need to step away from the stage director’s chair and get in the scene. But don’t get excited, you are not the protagonist! Let’s get serious, you can intervene when your students propose solutions that are unusual, incomplete or wrong. Here your intervention is crucial because it is precisely those moments that are key to the learners’ understanding. Since you are the one who knows about learning the most, you engage the whole class in the solution presented. Ask whether the other students agree or not, and why. Like this, your students will learn to analyze both their own and their classmates’ thoughts. Be patient, let the process do the job and don’t reveal the solution. They gradually come to draw conclusions from their past maths experiences, thus deepening their knowledge and understanding of mathematics.
Atmosphere creator
The atmosphere helps to bring excitement to your students and gets them in the mood for a Maths in Motion class. It is created in the relation between body, space and time and it appears to be a great motivator. Here are the basic elements for building this atmosphere: your own movements and rhythm, your voice and silence, the words you chose to use, the arrangement of the furniture in the space, the lighting and music. In order to get there, you simply need to immerse yourself into your role and be persistent. If you believe in you, your students will come along.

Coach
All the previous parts fall into coaching. So, all in all, you are a coach. A rather passive one, who doesn’t give solutions, but rather offers stimulus, questions, feedback and activities for reflection. To put it differently, you offer the space so the students get active and come up with their own solutions.

When it comes to working with groups, you are there to guide them to reach their goals. For example, in the “Dragons’ World” module in the “Storytelling” part (page 94). The teacher visits the groups to observe the progress made and in case the groups are stuck, they don’t give the students a ready story but supports them with questions to come up with their own story. Supportive questions could be: Who are the actors? Where are they? What are they doing?
How to work with the movement parts?
Hey, I am not a dancer!

Working with movement allows a completely different context to exist and at the beginning that can be confusing or a bit scary for the teacher. As a teacher, you are used to performing in a specific context with certain dynamics. Inviting students to learn in a Maths in Motion class, where they participate physically, can feel like inviting classroom chaos.

By context, we mean the setting of the classroom, the toolkits and/or tablets, pc, your position, the distance, the silence, individual or group work etc. All the context creates specific dynamics in the group of students and most of the time it requires stillness and quietness. A Maths in Motion class has a completely different dynamic: tables are gone, students are moving, you are moving, there is lots of dancing, laughing, discussing and there is music. These dynamics need a completely different skill-set to be managed, in order not to get out of hand.

The key to developing this skill-set is to allow yourself to shift from the role as teacher to the role of learner. Sometimes, you just need to prepare the atmosphere and give instructions, feedback, make interventions and facilitate reflection. Some other times it is required to dive into the process with the whole body while enjoying the experience. Like this, you are encouraging students to explore, create, and make sense of maths. Keep in mind that getting physically engaged supports both the introverts and shy students, as well as the hyperactive and noisy ones.

Let’s elaborate on this. When something new and unexpected is introduced, shyness and resistance can arise. There you use your own body and voice as instruments to encourage the students to move so they feel they’re being invited into the process. When everyone has joined and there are high levels of motion and energy, they won’t let it go and move on to the next activity. There you, being a fully engaged part of the process, can shape it in the direction you want. Don’t forget, you are always the organizer.

For example in the “Negative Numbers, Positive Learning” module, in the “warm and cold” part (page 105). The teacher starts moving with the students while there is “warm” music playing. With the movement of her body, the teacher is encouraging the class to move and she is staying connected with the students. The students are expected to move fast and intensely for some minutes. The teacher is part of them so when she stops, they also stop. What can also help is that the music stops at the same time.
How to work with the movement parts?

Hey, I am not a dancer!

In addition to the physical engagement that these modules invite you to make, there is also an invitation to engage one’s imagination and completely dive into different imaginary worlds. In most of the modules this “diving into a role” is proposed. Diving in a theatrical role for the teacher, and for the students, too.

The importance of these imaginary worlds and diving into them comes from the invitation to play. Even a physical effort is no more felt or experienced as a physical effort when we play and are inside “the game”, a kind of a movement lead play. What happens through this action is that one’s whole being is involved in the actions and thus we as a class of “game-participants” are even more credible while articulating our actions and more deep in the learning process. Through playing a role one’s body is involved fully in its whole kinesthetic sphere. This leads to a deeper participation as the leader and the students move through a universe that they have just created by making these modules theirs, by their interpretation.

This possibility to create your own role inside the framework, given by the module, gives a kind of liberty and possibility to include everyone in one’s own way. Engaging one’s kinesthetic sphere, on the other hand, means that we literally speak about dance - dancers’ knowledge and expertise in spatial perception.

Relating to mathematics which can be explored around and within us, means developing a capacity of engaging in a space larger than our own physical bodies while moving. This space is named kinesthetic space or sphere that we all have around our bodies. Acknowledging this not only gives more tools to the teachers to think of how to possibly lead a group and make them follow by the attraction that this larger body can create, but also increases the wholeness of physical engagement and often, the joy of it.

We believe that diving into the actions, truly and completely, also helps those who might not usually move in a class, desire to do so. This diving into a role, and creating a character while leading a module also, makes it such that the leader is on the same level with the students and everyone can “play the game” together - without the common hierarchical roles between learner and teacher. This possibility can be very refreshing and renew the group dynamics in the classes as well as renew teaching and learning experiences for all. Imagination and physical engagement are options to put maths into another dimension in our own bodies and in our non-verbal communication while moving with mathematical motivation.
The statistics presented in the toolkit are the research results, coming from the evaluations conducted during the project. The target group of the evaluations are: the experts who participated in the training activities of the Maths in Motion project and students who experienced Maths in Motion classes. The instruments used in both cases were 2 different questionnaires created and formed by Olde Vechte Foundation.

Specifically:
- Online evaluation conducted in June 2018 and the target was 15 teachers, movement and maths experts, STEAM educators and a learning specialist who participated in the training activities in Ommen, the Netherlands.
- Evaluation conducted in Athens, Greece, in May 2018 in the 1st Gymnasium of Kaisariani and the target was the 18 students who participated in the Get Angled class.
- Evaluation conducted in Athens, Greece, in May 2018 in the 1st Gymnasium of Kaisariani and the target was the 18 students who participated in the Dance of the Shape class.
- Evaluation conducted in Galati, Romania, in May 2018 in the Acoala Gimnaziul Nr 20 Galati and the target was the 19 students who participated in the Dragons’ World class.
- Evaluation conducted in Galati, Romania, in May 2018 in the Acoala Gimnaziul Nr 20 Balati and the target was the 21 students who participated in the Negative Numbers, Positive Learning class.
- Here it is important to make a clarification. The results which measure specific content information (e.g., problem solving) are presented separately and the name of the Maths in Motion class (e.g., Dragon’s World) is mentioned. The results which have general information (e.g., level of team spirit feelings), concern all the above mentioned students (76 in total), participated in all the Maths in Motion classes.

References


References

What do we consult?


PISA 2015 Results (Volume II) Summary in English: Excellence and Equity in Education


Illustrations

By Saara Lehto
Maths in Motion team is open for discussing or answering to any of your questions regarding to the content of the toolkit. Feel free to come in touch!

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